

# Proportional Representation



April 15, 2021

**POL 140A: Electoral Systems**

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**Spring Quarter, 2021**


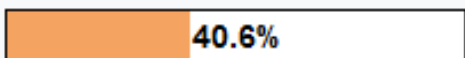


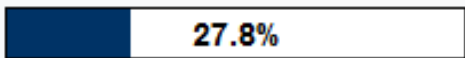
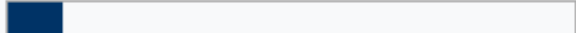

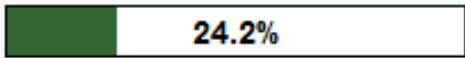


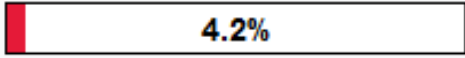
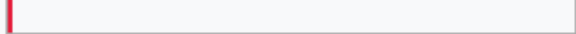


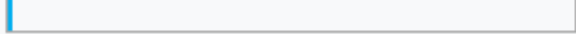
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# Outline

1. Alberta check-in!
2. Review: key concepts
3. The basics of PR
4. Allocating seats
5. D'Hondt
6. LR-Hare

# Checking in On Alberta!

# Practice at Home!: Alberta, 2015

Party		Votes			Seats	
	New Democratic	604,518	 40.6%	▲ 30.8pp	54 / 87 (62%) 	
	Progressive Conservative	413,610	 27.8%	▼ 16.2pp	9 / 87 (10%) 	
	Wildrose	360,511	 24.2%	▼ 10.1pp	21 / 87 (24%) 	
	Liberal	62,153	 4.2%	▼ 5.7pp	1 / 87 (1%) 	
	Alberta Party	33,221	 2.2%	▲ 0.9pp	1 / 87 (1%) 	

## OBSERVED

$$N_v = 3.30$$

$$N_s = 2.19$$

$$D_2 = 19.22$$

## PREDICTED

$$N_v = 2.54$$

$$N_s = 2.11$$

$$D_2 = 11.28$$

# Review: Key Concepts

# The Majoritarian-proportional Spectrum

Electoral systems are usually primarily classified according to their **proportionality** = *the degree to which they represent (groups of) voters equally - as opposed to over-representing the largest groups.*

## Majoritarian

## Proportional



- Plurality
- Runoff
- Alternative Vote (single-seat Ranked Choice Voting)

- Party-list Proportional (List PR)
- Single Transferable Vote (STV; multi-winner Ranked Choice Voting)

# Electoral Systems Type Matters

Electoral System I

Electoral System II

<b>% votes</b>	<b>% seats</b>	<b>% votes</b>	<b>% seats</b>
35.2	55.1	37.7	42.5
32.3	30.6	30	34.5
22.1	9.6	10.8	9.3
		10.1	7.1

“Plurality”

“Proportional”

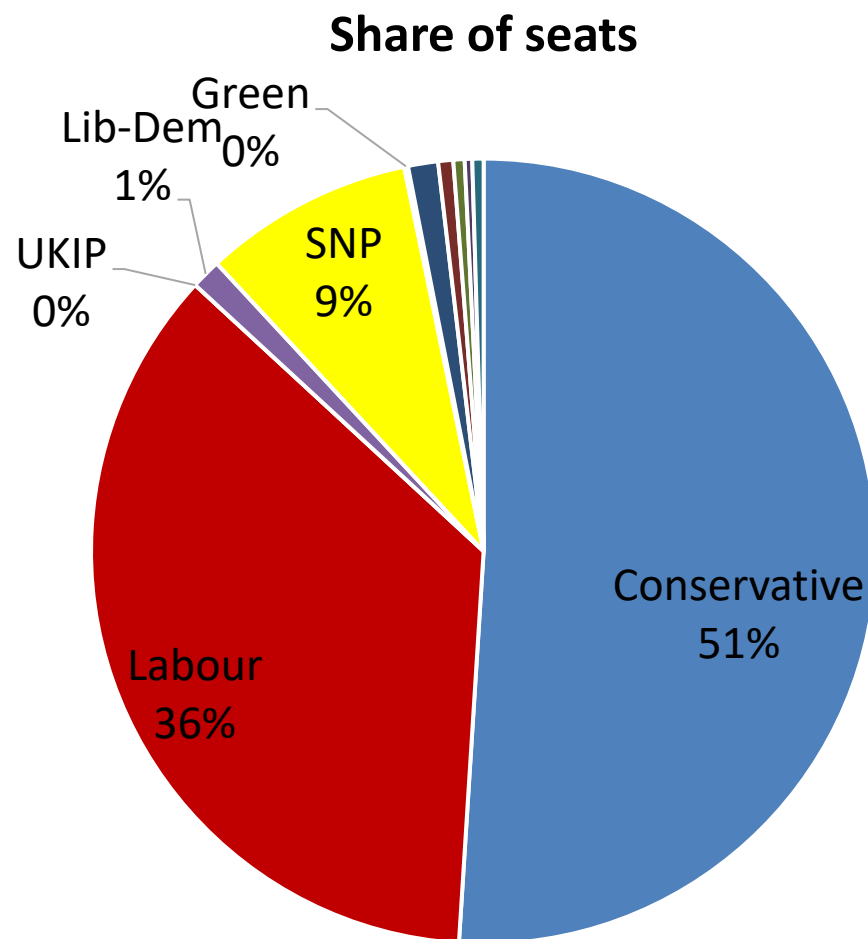
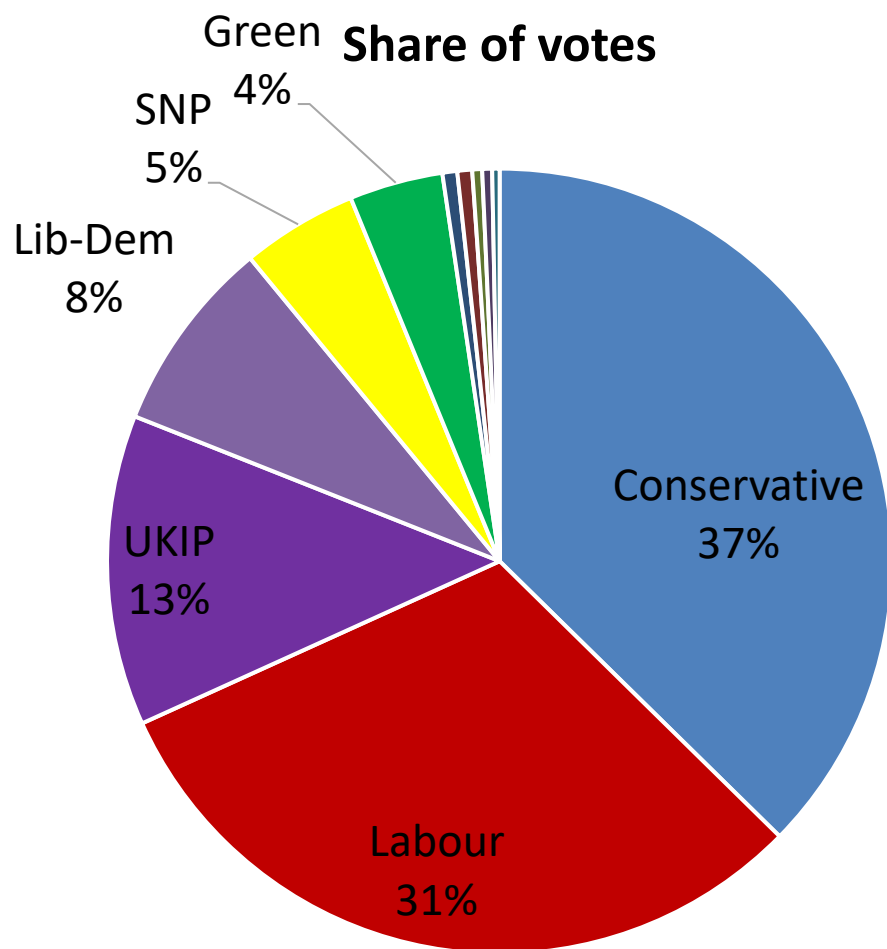
# Review: Key Concepts

- **Assembly size (S):** the total number of seats in the assembly
- **District magnitude (M):** the number of seats in a district
  - In US House districts, UK parliamentary constituencies, and Canadian ridings,  $M=1$ .
  - In national PR systems, like Israel,  $M=S$
  - In districted PR systems, like Portugal,  $M$  varies by district

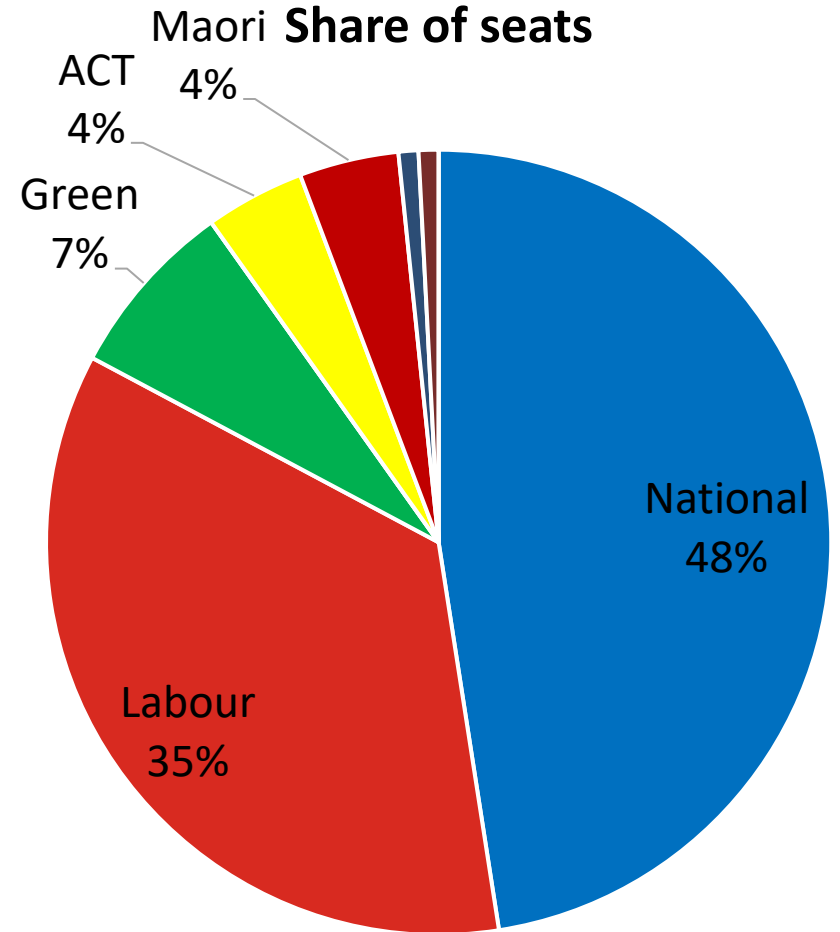
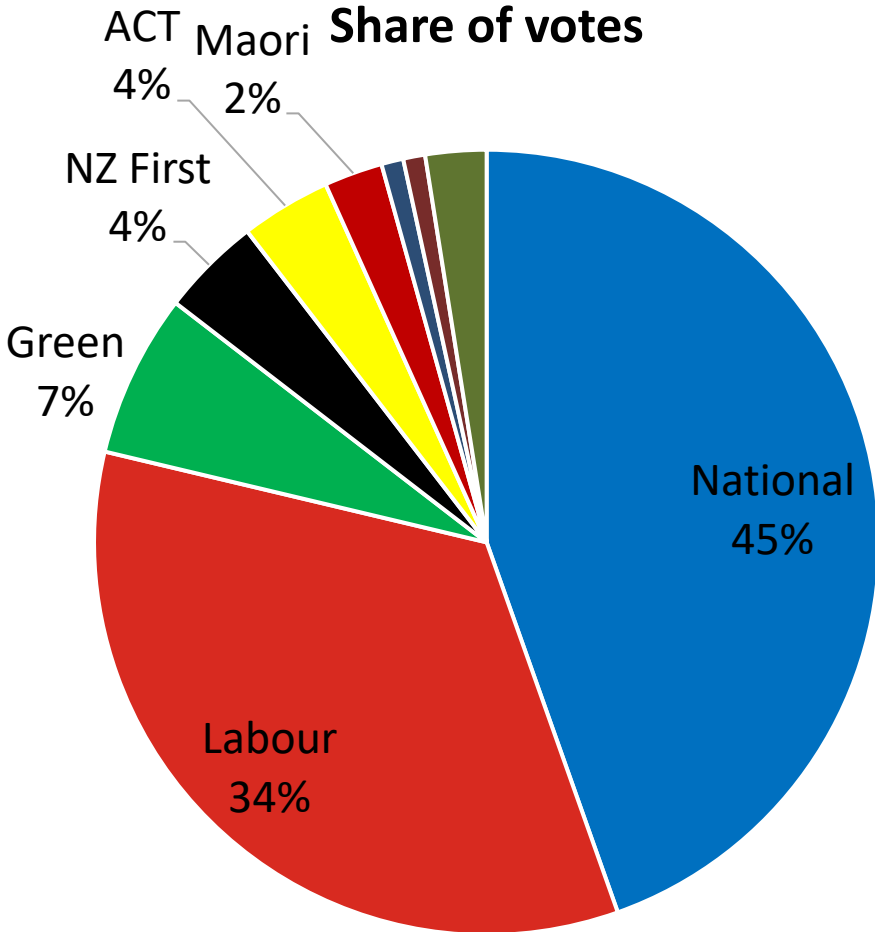


# The Basics of PR

# Typical Majoritarian Election: UK, 2005



# Typical PR Election: New Zealand, 2008

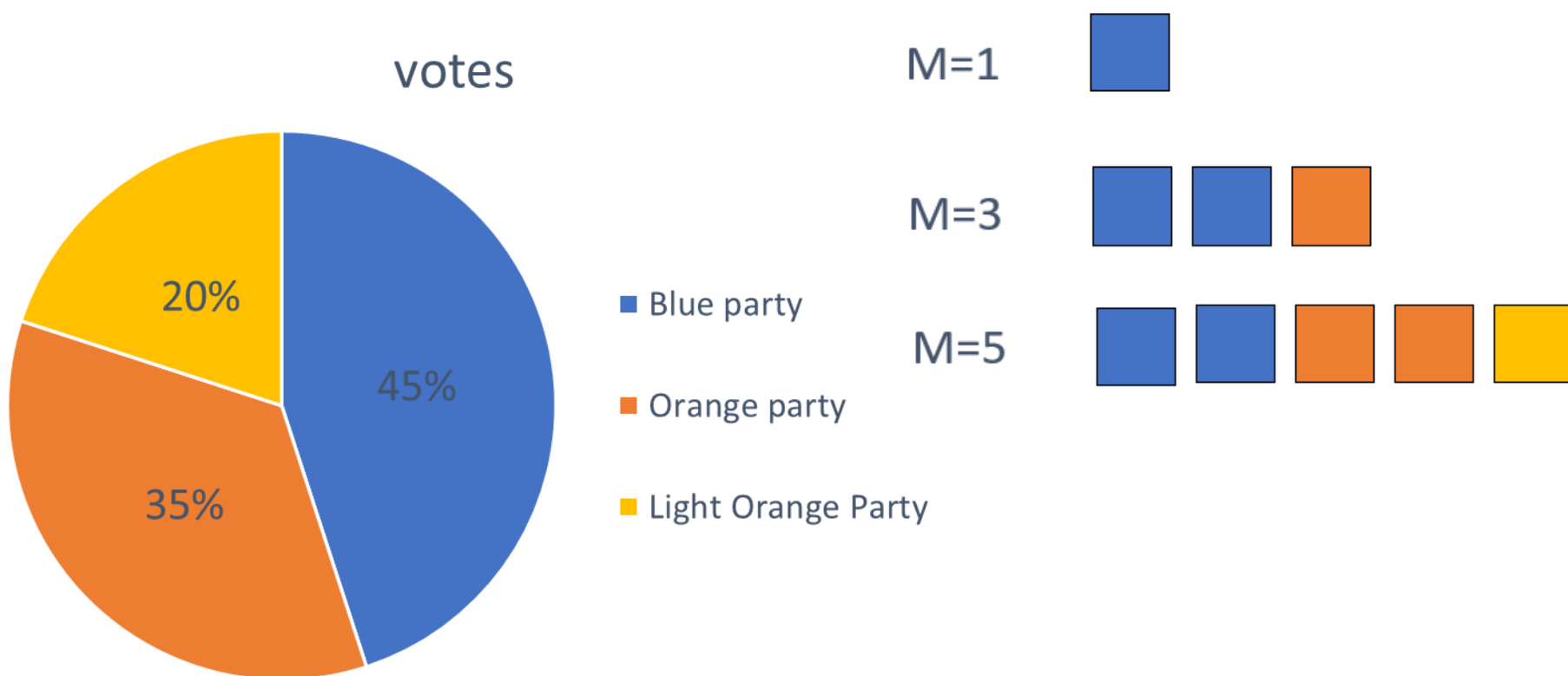


# Proportional Representation

- Electoral systems designed to represent parties (or other groups) according to their share of the total vote
- Under PR, a party winning 15% of the vote can usually expect to win about 15% of the seats
- **Must be multi-winner elections** (must use multi-seat districts) – single seats cannot be distributed among multiple parties!

# Majoritarian vs. Proportional Outcomes

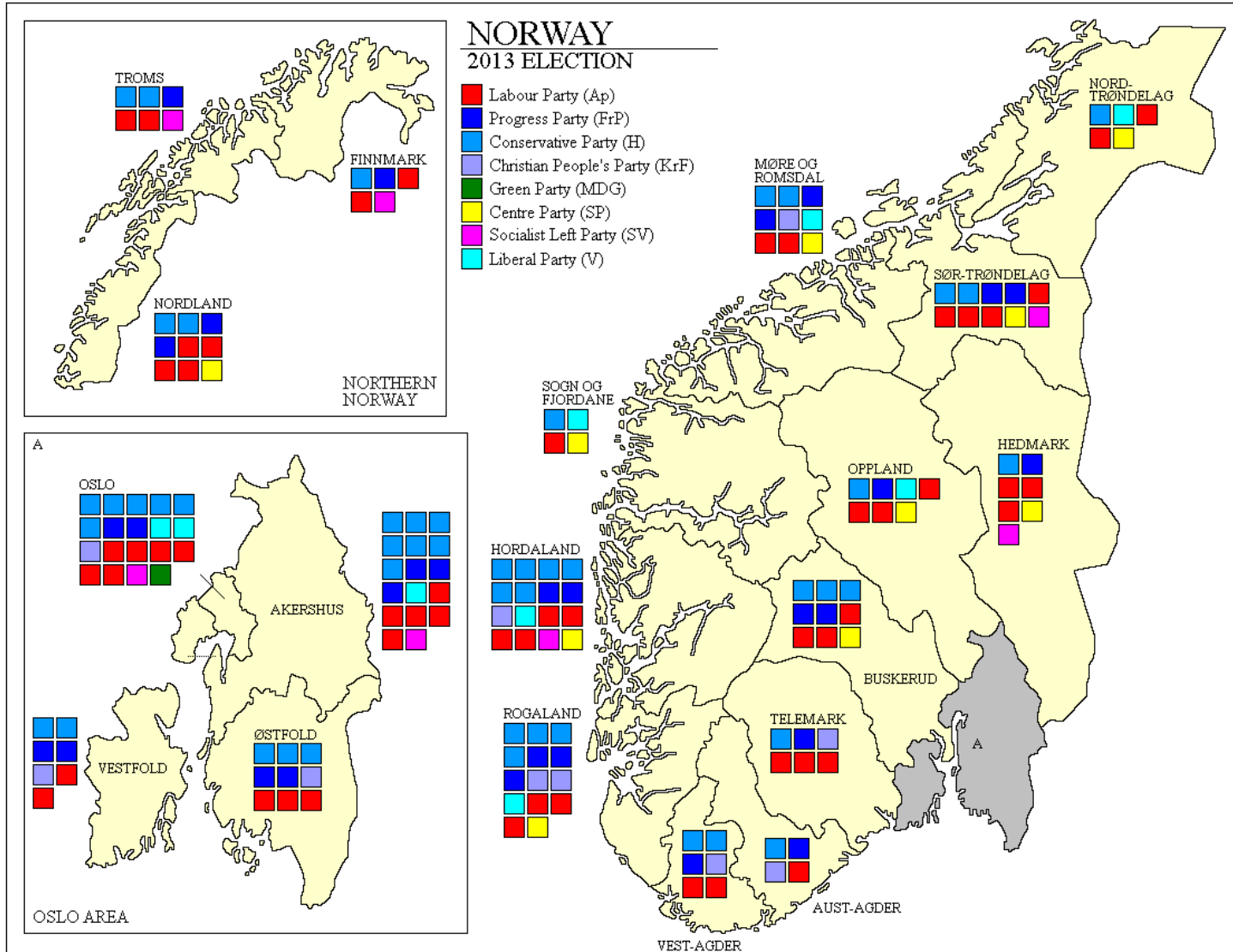
Reminder: District magnitude ( $M$ ) = the number of seats in a district



# Proportional Representation

- Votes are made for **party lists**. These lists can be either **closed** or **open**.
  - Each party puts forth a **list** of candidates
- Each party's list gets a % of seats **roughly** equal to its share of the vote
  - Electoral formulas are used to allocate seats proportionally
- Proportional representation can be national (M=S), as in Israel
- Often PR is districted, as in Belgium or Portugal

# Districted PR Example: Norway 2013



## Proportional Outcome Example: Norway (2013)

Party	Ideology	% votes	% seats
Labour	Center-left	30.8	32.5
Conservative	Center-right	26.8	28.4
Progress	Right	16.3	17.2
Christian Democratic	Center-right	5.6	5.9
Centre	Center	5.5	5.9
Liberal	Center	5.2	5.3
Socialist Left	Left	4.1	4.1
Green	Center-left	2.8	0.6
Others		2.7	0



# Proportional Outcome Example: Tunisia (2011)

Party	Ideology	% votes	% seats
Ennahda	Center-right (Islamist)	37.4	41.0
CPR	Leftist (secular)	8.7	13.4
Aridha	Populist	6.7	12.0
Ettakatol	Leftist (secular)	7.0	9.2
PDP	Center-left (secular)	3.9	7.4
Others		36.3	17.0

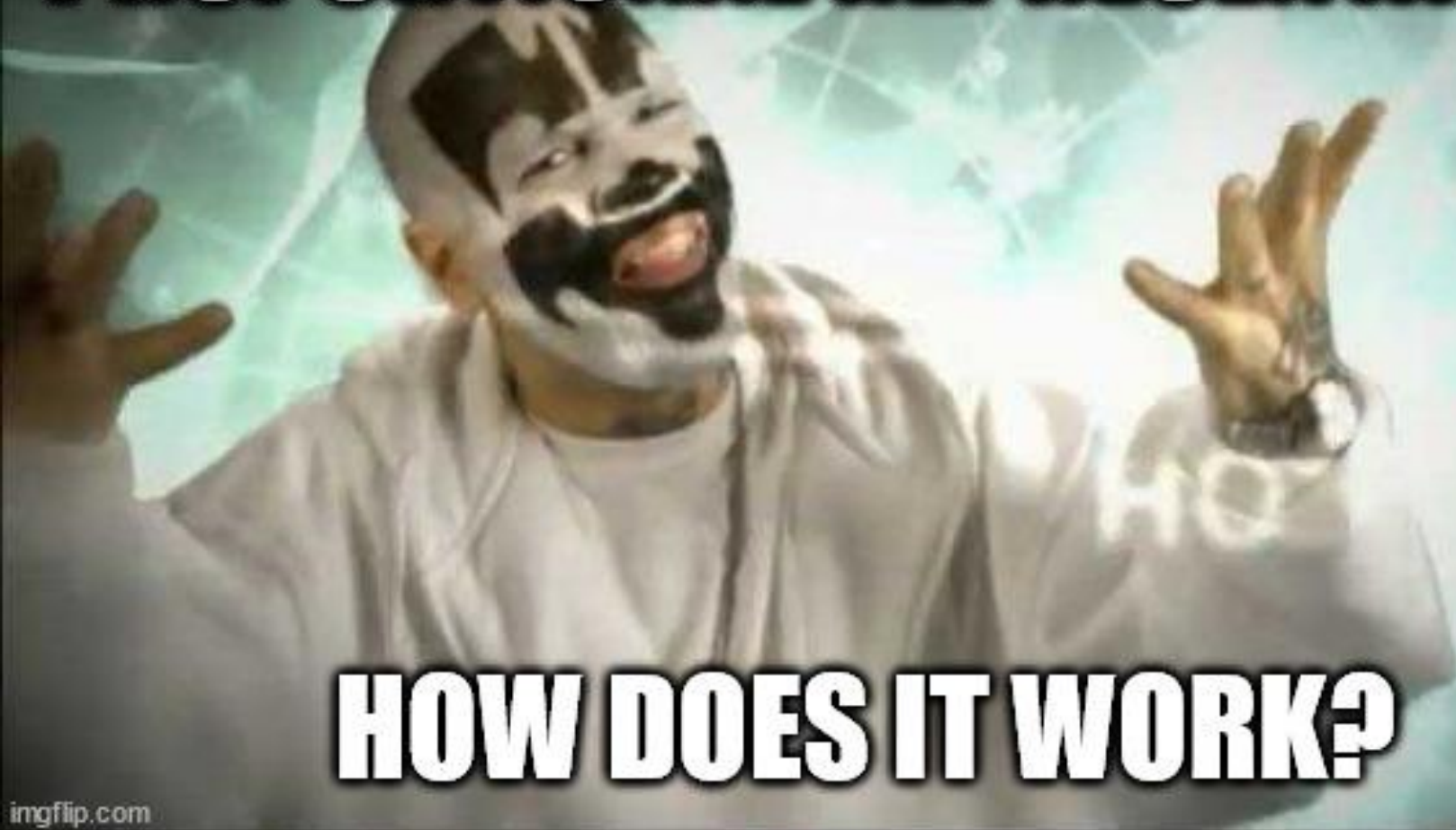
# CONTRAST: a “Normal” Majoritarian Election

## 2020 British Columbia Election

Party	% votes	% seats
BC NDP	45	63
BC Liberals	35	33
Green Party	15	3
Others	5	0

# Allocating Seats

**PROPORTIONAL REPRESENTATION**



**HOW DOES IT WORK?**

# The Basics of PR

- When using PR, seats are allocated by formula
- There are *many* such formulas (see Chapter 2!)
- We will focus on two simple & common ones
  1. D'Hondt
    - A divisor or “highest average” method
  2. Hare quota and largest remainders
    - AKA “simple quote”
    - Often abbreviated as “LR-Hare”

# D'Hondt

# D'Hondt

- Also known as the Jefferson method (as in Thomas Jefferson)
- The process:
  1. Count the total number of votes each party gets
  2. Determine M winners
- Compared to LR-Hare, D'hondt tends to favor the largest vote winner over smaller parties
  - Not nearly as much as a plurality system like FPTP!

# D'Hondt in Action

Party A	Party B	Party C	Party D	Party E
<b>38</b>	<b>31</b>	<b>17</b>	<b>9</b>	<b>5</b>

- Imagine that these are the results in a district following a PR election
- How do you allocate seats?
- If  $M=1$  who gets the first seat?
- Who gets the seats if  $M=2$ ?  $M=3$ ?



## D'Hondt *Divisors*

Divisor	Party A	Party B	Party C	Party D	Party E
<b>1</b>	<b>38</b>	<b>31</b>	<b>17</b>	<b>9</b>	<b>5</b>
<b>2</b>	19	15.5	8.5	4.5	2.5
<b>3</b>	12.7	10.3	5.7	3	
<b>4</b>	9.5	7.75	4.5		
<b>5</b>	7.6				
<b>6</b>	6.3				

- What have I done here?
  - Think back to chapter 2!

## D'Hondt *Divisors*

Divisor	Party A	Party B	Party C	Party D	Party E
<b>1</b>	<b>38 (1)</b>	<b>31</b>	<b>17</b>	<b>9</b>	<b>5</b>
<b>2</b>	<b>19</b>	<b>15.5</b>	<b>8.5</b>	<b>4.5</b>	<b>2.5</b>
<b>3</b>	<b>12.7</b>	<b>10.3</b>	<b>5.7</b>	<b>3</b>	
<b>4</b>	<b>9.5</b>	<b>7.75</b>	<b>4.5</b>		
<b>5</b>	<b>7.6</b>				
<b>6</b>	<b>6.3</b>				

- Party A gets the first seat. Why?
- If  $M > 1$ , who gets the second seat? Why?

## D'Hondt *Divisors*

Divisor	Party A	Party B	Party C	Party D	Party E
<b>1</b>	<b>38 (1)</b>	<b>31 (2)</b>	<b>17</b>	<b>9</b>	<b>5</b>
<b>2</b>	<b>19</b>	<b>15.5</b>	<b>8.5</b>	<b>4.5</b>	<b>2.5</b>
<b>3</b>	<b>12.7</b>	<b>10.3</b>	<b>5.7</b>	<b>3</b>	
<b>4</b>	<b>9.5</b>	<b>7.75</b>	<b>4.5</b>		
<b>5</b>	<b>7.6</b>				
<b>6</b>	<b>6.3</b>				

- Seat 2 goes to Party B
  - Notice I have crossed out Party A's 1<sup>st</sup> divisor!
- If  $M > 2$ , who gets seat 3?

## D'Hondt *Divisors*

Divisor	Party A	Party B	Party C	Party D	Party E
1	38 <b>(1)</b>	31 <b>(2)</b>	17	9	5
2	19 <b>(3)</b>	15.5	8.5	4.5	2.5
3	12.7	10.3	5.7	3	
4	9.5	7.75	4.5		
5	7.6				
6	6.3				

- Seat 3 goes to Party A
  - Why not to Party C? They came in 3<sup>rd</sup>!
- If  $M > 3$ , who gets seat 4?

## D'Hondt *Divisors*

Divisor	Party A	Party B	Party C	Party D	Party E
1	38 <b>(1)</b>	31 <b>(2)</b>	17 <b>(4)</b>	9	5
2	19 <b>(3)</b>	15.5	8.5	4.5	2.5
3	12.7	10.3	5.7	3	
4	9.5	7.75	4.5		
5	7.6				
6	6.3				

- Seat 4 goes to Party C
- We could do this all day!

Divisor	Party A	Party B	Party C	Party D	Party E
1	38 (1)	31 (2)	17 (4)	9 (9)	5 (18)
2	19 (3)	15.5 (5)	8.5 (10)	4.5	2.5
3	12.7 (6)	10.3 (7)	5.7 (15)	3	
4	9.5 (8)	7.75 (11)	4.25	2.25	
5	7.6 (12)	6.2 (14)	3.4		
6	6.3 (13)	5.17 (17)			
7	5.4 (16)	4.43			
<b>M=4</b>	2	1	1	0	0
<b>M=9</b>	4	3	1	1	0
<b>M=18</b>	7	6	3	1	1

# As M Increases, so Does Proportionality

% votes:

<b>38</b>	<b>31</b>	<b>17</b>	<b>9</b>	<b>5</b>
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<b>M=4</b>	<b>2</b> <b>(50.0%)</b>	<b>1</b> <b>(25.0%)</b>	<b>1</b> <b>(25.0%)</b>	<b>0</b>	<b>0</b>
<b>M=9</b>	<b>4</b> <b>(44.4%)</b>	<b>3</b> <b>(33.3%)</b>	<b>1</b> <b>(11.1%)</b>	<b>1</b> <b>(11.1%)</b>	<b>0</b>
<b>M=18</b>	<b>7</b> <b>(38.8%)</b>	<b>6</b> <b>(33.3%)</b>	<b>3</b> <b>16.7%</b>	<b>1</b> <b>(5.6%)</b>	<b>1</b> <b>(5.6%)</b>

# D'Hondt in Action





# LR-Hare

# The Basics of LR-Hare

- Also known as the “Hamilton method”
1. You start by calculating a **quota** – how many votes are required to win a seat:

$$\text{Hare quota} = \frac{\textit{Total Vote}}{\textit{Total Seats}} = \frac{\textit{Total Vote}}{M}$$

2. You then divide each party’s votes by the quota, and award seats based on the result
- Compared to D’Hondt, this method is more favorable to smaller parties

# LR-Hare in Action

Party A	Party B	Party C	Party D	Party E
<b>38</b>	<b>31</b>	<b>17</b>	<b>9</b>	<b>5</b>

- Do these results look familiar?
- Let's suppose:
  - 100 votes
  - $M=5$
- What is the Hare quota for this election?

# LR-Hare in Action

Party A	Party B	Party C	Party D	Party E
38	31	17	9	5

- With 100 votes and  $M=5$ , the Hare quota is  $100/5 = 20$
- How many full quotas can we assign?
- OK, so Parties A and B get one seat each...
- How do we allocate the remaining 3?
- This is the “largest remainders” part!

# LR-Hare in Action

- The Hare quota is **20**

	Party A	Party B	Party C	Party D	Party E	
Votes	<b>38</b>	<b>31</b>	<b>17</b>	<b>9</b>	<b>5</b>	<b>Seats at stage</b>
Quotas	<b>1</b>	<b>1</b>	<b>0</b>	<b>0</b>	<b>0</b>	<b>2</b>
Remainder	$38-(1*20)=$ <b>18</b>	$31-(1*20)=$ <b>11</b>	$17-(0*20)=$ <b>17</b>	<b>9</b>	<b>5</b>	<b>3</b>
Remainder seats	<b>?</b>	<b>?</b>	<b>?</b>	<b>?</b>	<b>?</b>	
Total seats						

# LR-Hare in Action

- The Hare quota is **20**

	Party A	Party B	Party C	Party D	Party E	
Votes	<b>38</b>	<b>31</b>	<b>17</b>	<b>9</b>	<b>5</b>	Seats at stage
Quotas	<b>1</b>	<b>1</b>	<b>0</b>	<b>0</b>	<b>0</b>	<b>2</b>
Remainder	$38-(1*20)=$ <b>18</b>	$31-(1*20)=$ <b>11</b>	$17-(0*20)=$ <b>17</b>	<b>9</b>	<b>5</b>	<b>3</b>
Remainder seats	<b>1</b>	<b>1</b>	<b>1</b>	<b>0</b>	<b>0</b>	
Total seats	<b>2</b>	<b>2</b>	<b>1</b>	<b>0</b>	<b>0</b>	

# LR-Hare in Action

- Let's imagine now that  $M=10$  (still 100 voters)
- What is the quota?
- $100/10 = 10$

	Party A	Party B	Party C	Party D	Party E	
Votes	38	31	17	9	5	Seats at stage
Quotas	?	?	?	?	?	?
Remainder	?	?	?	?	?	?
Remainder seats	?	?	?	?	?	
Total seats	?	?	?	?	?	

# LR-Hare in Action

- Let's imagine now that  $M=10$  (still 100 voters)
- What is the quota?
- $100/10 = 10$

	Party A	Party B	Party C	Party D	Party E	
Votes	38	31	17	9	5	Seats at stage
Quotas	3	3	1	0	0	7
Remainder	?	?	?	?	?	?
Remainder seats	?	?	?	?	?	
Total seats	?	?	?	?	?	



# LR-Hare in Action

- Let's imagine now that  $M=10$  (still 100 voters)
- What is the quota?
- $100/10 = 10$

	Party A	Party B	Party C	Party D	Party E	
Votes	<b>38</b>	<b>31</b>	<b>17</b>	<b>9</b>	<b>5</b>	<b>Seats at stage</b>
Quotas	<b>3</b>	<b>3</b>	<b>1</b>	<b>0</b>	<b>0</b>	<b>7</b>
Remainder	$38 - (3 \cdot 10) =$ <b>8</b>	$31 - (3 \cdot 10) =$ <b>1</b>	$17 - (1 \cdot 10) =$ <b>7</b>	<b>9</b>	<b>5</b>	<b>3</b>
Remainder seats	<b>?</b>	<b>?</b>	<b>?</b>	<b>?</b>	<b>?</b>	
Total seats	<b>?</b>	<b>?</b>	<b>?</b>	<b>?</b>	<b>?</b>	

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- Let's imagine now that  $M=10$  (still 100 voters)
- What is the quota?
- $100/10 = 10$

	Party A	Party B	Party C	Party D	Party E	
Votes	38	31	17	9	5	Seats at stage
Quotas	3	3	1	0	0	7
Remainder	8	1	7	9	5	3
Remainder seats	1	0	1	1	0	
Total seats	?	?	?	?	?	

# LR-Hare in Action

- Let's imagine now that  $M=10$  (still 100 voters)
- What is the quota?
- $100/10 = 10$

	Party A	Party B	Party C	Party D	Party E	
Votes	<b>38</b>	<b>31</b>	<b>17</b>	<b>9</b>	<b>5</b>	<b>Seats at stage</b>
Quotas	<b>3</b>	<b>3</b>	<b>1</b>	<b>0</b>	<b>0</b>	<b>7</b>
Remainder	<b>8</b>	<b>1</b>	<b>7</b>	<b>9</b>	<b>5</b>	<b>3</b>
Remainder seats	<b>1</b>	<b>0</b>	<b>1</b>	<b>1</b>	<b>0</b>	
Total seats	<b>4</b>	<b>3</b>	<b>2</b>	<b>1</b>	<b>0</b>	

# Questions?